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Conjugate Leveque solution for Newtonian fluid in a parallel plate channel

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Abstract—Leveque solution for conjugate problem of high Prandtl number, Newtonian fluid flowing in a finite length, parallel plate channel is presented. A procedure is proposed to find the approximate interfacial temperature distribution. It is found that the effect of wall conduction can be characterized by two parameters. A close form, approximate solution for local Nusselt number is obtained. Although the solution presented in this work is supposed to be valid only for $x^+ \leqslant 0.001$, results show that it does not deviate significantly from the known eigenfunction expansion solution even when $x^+ = 0.005$.

INTRODUCTION

ALL CONVECTIVE heat transfer problems are actually conjugate problems which treat the solid wall and fluid as an integral system. Traditionally, most convective heat transfer problems are treated by solving the energy equation of the fluid phase alone, imposing the boundary conditions at the solid–fluid interface. This is equivalent to neglecting the effect of wall resistance; unless the wall thermal resistance is small, the solution will be in error. Little work has been done on solid–fluid conjugate problems as compared to the classical convective heat transfer problems when wall resistance is neglected. Shah and London [1] gave a brief review of works on conjugate problems prior to 1976. Recent works on conjugate problems were described in detail by Barozzi and Pagliarini [2]. Theoretical analyses on the effect of wall conduction on the rate of heat transfer for laminar flow of Newtonian fluid flowing in a pipe or parallel plate channel include that of Aleksashenko [3] and that of Luikov *et al.* [4]. They treated the problem as a semi-infinite duct. The solutions are expressed as a combination of complicated functions, integrals and infinite series. No numerical results are available to compare with the traditional convective heat transfer problem under the same boundary conditions. The works of Mori *et al.* [5, 6] are by far the most extensive analytical studies on conjugate heat transfer. Their conclusions include: (1) the local Nusselt number of conjugate problem falls between that of constant wall temperature and constant wall flux when axial conduction along the wall is neglected; and (2) in the case of constant heat flux at the outer wall, the effect of axial conduction along the wall is to decrease the local Nusselt number to a value closer to that of constant wall temperature when axial conduction along the wall is neglected. On the other hand, when the outer wall boundary condition is isothermal, the effect of axial conduction along the wall is to increase the local Nusselt number to a value closer to that of constant wall flux when axial conduction along the wall is neglected. The conjugate problem of Poiseuille–Couette flow between parallel flat plates was treated by Davis and Gill [7]. When constant heat flux was imposed at the outer wall, their results agree closely with that of Mori *et al.* [6]. A general description of conjugate problems with examples in heat and mass transfer applications was given by Davis and Venkatesh [8]. Solutions were obtained by using integral equation formulations.

For low Péclet number convective heat transfer, the effect of axial conduction must be included in fluid as well as in solid phases. The energy equations for both phases are elliptic and more difficult to solve than the case when fluid axial conduction is neglected. Analytical solutions of the conjugate problem with axial conduction include that of Papoutsakis and Ramkrishna [9], Ju and Lee [10]. In principle, the solutions are expressed as an infinite series of eigenfunctions. The crux of the solution is to apply the matching principle at the solid–fluid interface which requires that both the temperature and the heat flux be continuous.

Numerical methods have been applied by some investigators to solve conjugate problems. Fahri and Sparrow [11] treated a pipe as having infinite domain and assumed the thickness of the wall to be small so that the energy equation for the solid phase reduced to a one-dimensional heat conduction equation. Barozzi and Pagliarini [2] solved the conjugate heat transfer problem of a heated section of pipe of finite length by using a combination of the finite-element method at the wall and the superposition principle at the interface.

The use of the eigenfunction expansion technique in the analytical solution of conjugate problems has limitations in application. When the axial distance is small or the Prandtl number is large, the convergence of the series solution is slow and instability may arise in actual numerical calculation. This is why the results of both Mori *et al.* [5, 6] and Davis and Gill [7] are

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NOMENCLATURE

A_j coefficient defined in equation (17)
$B(i, j)$ beta function
b half depth of parallel plate channel
C_p specific heat of fluid
k thermal conductivity
L length of parallel plate channel
Nu_x local Nusselt number
p pressure
Pe Péclet number, $4bu_m/\alpha$
Pr Prandtl number
Q_w dimensionless q_w, $q_w L/k_s T_0$
q_w heat flux at outer wall
q'' heat flux at interface
T temperature
u axial velocity of fluid
x axial coordinate
x^* dimensionless axial coordinate, x/L
$x^\dagger$ $x/(4bPe)$
y coordinate normal to x
y^* dimensionless y, y/L.

Greek symbols
α thermal diffusivity of fluid
λ dummy variable
μ viscosity
ρ density of fluid
$\Gamma(\ )$ gamma function
θ dimensionless temperature
τ_w wall shear stress
ξ dimensionless dummy variable
δ thickness of the flat plate
Δ parameter defined in equation (14).

Subscript
f fluid
i interface
m bulk mean value
0 inlet
s solid
w wall
x local value at position x

applicable only to low Prandtl number gases. For fluids with a large Prandtl number, the diffusion of heat is limited to a thin layer (called the boundary layer) near the wall and can be predicted. Stewart [12–14] applied this principle to solve three-dimensional heat transfer problems. In this paper, results of the effect of wall resistance on the rate of heat transfer of high Prandtl number, Newtonian fluid flowing between parallel plates are presented.

ANALYSIS AND SOLUTION

Figure 1 is a schematic description of the conjugate heat transfer problem considered in this work. Since the fluid considered in this work has a high Prandtl number, the velocity at the inlet can be assumed to have a fully developed profile. Fluid is assumed to enter the channel with a uniform temperature T_0. For the convenience of analysis, both end surfaces of the parallel plate are assumed insulated. Under assumptions of steady state and constant physical properties, energy equations for both phases are linear and are coupled through boundary conditions at the interface which require that both the temperature and the heat flux be continuous. Energy equations for the fluid phase and solid phase are

$$\rho C_p u(\partial T_f/\partial x) = k_f(\partial^2 T_f/\partial y^2) \quad (1)$$

$$\partial^2 T_s/\partial x^2 + \partial^2 T_s/\partial y^2 = 0 \quad (2)$$

with boundary conditions

$$T_f = T_0 \quad \text{at} \quad x = 0, \quad 0 < y < 2b \quad (3)$$

$$\left.\begin{aligned} T_f = T_s = T_i(x) \quad (4a) \\ -k_f\,\partial T_f/\partial y = -k_s\,\partial T_s/\partial y \quad (4b) \end{aligned}\right\} \text{at} \quad y = 0, \quad 0 < x < L$$

$$\partial T_s/\partial x = 0 \quad \text{at} \quad x = 0 \quad \text{and} \quad x = L, \quad -\delta < y < 0. \quad (5)$$

For uniform wall temperature (UWT)

$$T_s = T_w \quad \text{at} \quad y = -\delta, \quad 0 < x < L. \quad (6a)$$

For uniform heat flux (UHF)

$$-k_s\,\partial T_s/\partial y = q_w \quad \text{at} \quad y = -\delta, \quad 0 < x < L. \quad (6b)$$

Note that both boundary conditions (6a) and (6b) are imposed at the outside surface of the wall. Because of symmetry, only the lower half of the duct has been considered. Energy equations (1) and (2) are coupled through boundary conditions (4a) and (4b) which means that both temperature and heat flux are continuous at the interface. This is called a conjugate

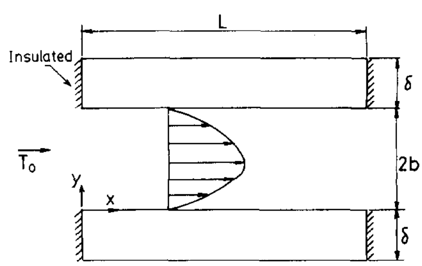

FIG. 1. Schematic diagram of the problem and the coordinate system.

problem. For fluids with a high Prandtl number, the thermal boundary layer is very thin near the channel entrance and velocity of fluid in the thermal boundary layer can be approximated by a linear function of transverse distance y

$$u = (\tau_w/\mu)y + O(y^2). \tag{7}$$

If higher accuracy is needed the term $(1/2\mu)(dp/dx)y^2$ can be added to the RHS of equation (7) and the error will be $O(y^3)$. If the wall resistance is neglected, the solution of equation (1) with the velocity given by equation (7) and boundary condition of constant wall temperature can be expressed as

$$\frac{T_f - T_w}{T_0 - T_w} = \frac{1}{\Gamma(4/3)} \int_0^{\eta} \exp(-t^3)\,dt \tag{8}$$

where

$$\eta = \frac{y}{b}(u_m b^2/3\alpha x)^{1/3}.$$

Since equation (8) is a solution of boundary-layer type, it satisfies $T_f = T_0$ as $y \to \infty$. The local Nusselt number that corresponds to temperature distribution given by equation (8) is

$$Nu_x = 1.233/(x^+)^{1/3}. \tag{9}$$

This is the famous Leveque solution which is an accurate asymptotic solution for $x^+ = x/(4bPe) < 0.001$. However, due to the effect of wall conduction, the solid–fluid interface temperature will no longer be constant even when the outside wall temperature is held constant. The interface temperature will in general be function of x. By the application of Duhamel's theorem, the fluid temperature can be expressed as

$$T_f = T_0 + \int_0^x [T_i(\lambda) - T_0] \times \frac{\partial}{\partial x}\left[1 - \frac{1}{\Gamma(4/3)}\int_0^{\eta(x-\lambda,y)} \exp(-t^3)\,dt\right] d\lambda. \tag{10}$$

Equation (10) is transformed into dimensionless form as

$$\theta_f = 1 + \int_0^{x^*} [\theta_i(\xi) - 1] \times \frac{\partial}{\partial x^*}\left[1 - \frac{1}{\Gamma(4/3)}\int_0^{\eta(x^*-\xi,y^*)} \exp(-t^3)\,dt\right] d\xi \tag{11}$$

where

$$x^* = x/L, \quad y^* = y/L$$
$$\theta = T/T_0 \qquad \text{(UHF)}$$
$$\theta = (T - T_w)/(T_0 - T_w) \quad \text{(UWT)}.$$

Temperature distribution in the solid phase can be found by the method described in Carslaw and Jaeger [15]. The temperature distribution for solid phase with boundary condition of UHF and UWT is

$$\theta_s = 2\sum_{n=1}^{\infty} \frac{\cosh n\pi(y^* + \delta/L)}{\cosh n\pi\delta/L} \times \cos n\pi x^* \int_0^1 \theta_i(\xi) \cos n\pi\xi\,d\xi + \int_0^1 \theta_i(\xi)\,d\xi - Q_w y^* \quad \text{(UHF)} \tag{12a}$$

$$\theta_s = 2\sum_{n=1}^{\infty} \frac{\sinh n\pi(y^* + \delta/L)}{\sinh n\pi\delta/L} \times \cos n\pi x^* \int_0^1 \theta_i(\xi) \cos n\pi\xi\,d\xi + \left(\frac{y^*}{\delta/L} + 1\right)\int_0^1 \theta_i(\xi)\,d\xi \quad \text{(UWT)}. \tag{12b}$$

In equations (11) and (12), $\theta_i(x^*)$ remains as an unknown which should be determined by applying the principle of continuity of heat flux at the solid interface, i.e. equation (4b). By substituting equations (11) and (12) into equation (4b) we have for the case of the UHF boundary condition

$$\frac{1}{\Delta(12)^{1/3}\Gamma(4/3)} \int_0^{x^*} [\theta_i(\xi) - 1] \frac{\partial}{\partial x^*}(x^* - \xi)^{-1/3}\,d\xi = Q_w - 2\sum_{n=1}^{\infty} n\pi \tanh n\pi\frac{\delta}{L} \cos n\pi x^* \times \int_0^1 \theta_i(\xi) \cos n\pi\xi\,d\xi \tag{13a}$$

and for the case of UWT boundary condition

$$\frac{1}{\Delta(12)^{1/3}\Gamma(4/3)} \int_0^{x^*} [\theta_i(\xi) - 1] \frac{\partial}{\partial x^*}(x^* - \xi)^{-1/3}\,d\xi = -\frac{1}{\delta/L}\int_0^1 \theta_i(\xi)\,d\xi - 2\sum_{n=1}^{\infty} n\pi \coth n\pi\frac{\delta}{L} \cos n\pi x^* \int_0^1 \theta_i(\xi) \cos n\pi\xi\,d\xi \tag{13b}$$

where the parameter Δ is defined as

$$\Delta = (b/L)^{2/3} Pe^{-1/3}(k_s/k_f). \tag{14}$$

By applying Abel's transformation formula [16], equations (13a) and (13b) are transformed into equations (15a) and (15b) respectively

$$\theta_i(x^*) - 1 = \frac{\Gamma(4/3)(12)^{1/3}\sin\pi/3}{\pi}\Delta \times \left\{3Q_w x^{*1/3} - 2\sum_{n=1}^{\infty}\left[\int_0^1 \theta_i(\xi)\cos n\pi\xi\,d\xi\right] \times n\pi \tanh n\pi\frac{\delta}{L}\int_0^{x^*}\frac{\cos n\pi s}{(x^* - s)^{2/3}}\,ds\right\} \tag{15a}$$

$$\theta_i(x^*)-1 = \frac{\Gamma(4/3)(12)^{1/3}\sin\pi/3}{\pi}\Delta \times\left\{-\frac{3x^{*1/3}}{\delta/L}\int_0^1 \theta_i(\xi)\,d\xi -2\sum_{n=1}^{\infty}\left[\int_0^1 \theta_i(\xi)\cos n\pi\xi\,d\xi\right] \times n\pi\coth n\pi\frac{\delta}{L}\int_0^{x^*}\frac{\cos n\pi s}{(x^*-s)^{2/3}}\,ds\right\}. \quad (15b)$$

Since $\theta_i(x^*)$ appears both in the RHS and the LHS of equations (15a) and (15b), an explicit form of $\theta_i(x^*)$ is not available. We use an iteration method to find $\theta_i(x^*)$ using the wall temperature obtained when wall resistance is neglected as an initial guess, i.e.

$$\theta_i^{(0)}(x^*) = 1+\frac{3\Gamma(4/3)(12)^{1/3}\sin\pi/3}{\pi}\Delta Q_w x^{*1/3} \quad \text{(UHF)} \quad (16a)$$

$$\theta_i^{(0)}(x^*) = 1 \quad \text{(UWT)}. \quad (16b)$$

Equations (16a) and (16b) are substituted into the RHS of equations (15a) and (15b) to obtain $\theta_i^{(1)}(x^*)$ in the LHS. This procedure is continued until the required accuracy is met. If the wall resistance is small, i.e. if δ/L or Δ is small, it needs only one or two iterations to obtain a satisfactory $\theta_i(x^*)$. However, for large values of δ/L or Δ, the iteration converges too slowly to be practical. In this case, an alternative procedure is used [17]. From equations (15a) and (15b), it seems that $\theta_i(x^*)$ can be expressed as a power series of $x^{*1/3}$. It is assumed that $\theta_i(x^*)$ can be expressed as

$$\theta_i(x^*)-1 = \sum_{j=0}^{N} A_j x^{*j/3}. \quad (17)$$

From equations (13a) and (13b), we see that the RHSs are in the form of a Fourier cosine series. As an example, if equation (17) is substituted into (13a), we have

$$\frac{1}{\Delta(12)^{1/3}\Gamma(4/3)}\times\left[A_0x^{*-1/3}+\sum_{j=1}^{N}A_j\frac{j}{3}B\left(\frac{2}{3},\frac{j}{3}\right)x^{*(j-1)/3}\right] = Q_w-2\sum_{n=1}^{\infty}\left[n\pi\tanh n\pi\frac{\delta}{L}\cos n\pi x^* \times\int_0^1\left(1+\sum_{j=0}^{N}A_j\xi^{j/3}\right)\cos n\pi\xi\,d\xi\right]. \quad (18)$$

Multiply both sides of equation (18) by $\cos j\pi x^*, j = 0, 1, 2, \ldots, N$ and integrate with respect to x^* from 0 to 1, we have $N+1$ linear algebraic equations in $N+1$ unknowns $A_0, A_1, A_2, \ldots, A_N$ which can be solved easily by Gauss elimination.

After obtaining the interfacial temperature profile, we can proceed to calculate the fluid temperature and hence the rate of heat transfer. The interface local Nusselt number is defined as

$$Nu_x = \frac{4bq_f''/k_f}{T_i-T_0}. \quad (19)$$

The relation between Nu_x and $\theta_i(x^*)$ is

$$Nu_x = \frac{(16/3)^{1/3}}{\Gamma(4/3)}\frac{(b/L)(k_s/k_f)}{\Delta}\times\left\{\int_0^{x^*}[\theta_i(\xi)-1]\frac{\partial}{\partial x^*}(x^*-\xi)^{-1/3}\,d\xi\right\}\Big/[\theta_i(x^*)-1]. \quad (20)$$

Or by substituting equation (17) into (20), we have

$$Nu_x = \frac{(4/3)^{1/3}}{\Gamma(4/3)}\left[A_0x^{\dagger-1/3}+\sum_{j=1}^{N}\frac{j}{3}B\left(\frac{2}{3},\frac{j}{3}\right)\times A_j\left(4\frac{b}{L}Pe\right)^{j/3}x^{\dagger(j-1)/3}\right]\Big/[\theta_i(x^*)-1]. \quad (21)$$

If only A_0 is present in equation (17), then equation (21) reduces to the classical Leveque solution for constant wall temperature, i.e. equation (9). If only A_1 is present, equation (21) reduces to

$$Nu_x = 1.490/x^{\dagger 1/3} \quad (22)$$

which is the classical Leveque solution for constant wall flux.

RESULTS AND DISCUSSION

From the analysis presented in the previous section, we see that even for a simple conjugate problem as that discussed in this work, the solutions are much more complicated than the classical convective heat transfer problems. The local Nusselt number for the classical Leveque solution is a function of dimensionless axial distance only. For the conjugate problem described in this work it is apparent, from equations (15a), (15b) and (20), that the local Nusselt number depends on four parameters and can be expressed as

$$Nu_x = \frac{b}{L}\frac{k_s}{k_f}fn(\Delta, x/L, \delta/L)$$

or

$$Nu_x^* = Nu_x\Big/\left(\frac{b}{L}\frac{k_s}{k_f}\right) = fn(\Delta, x/L, \delta/L).$$

In the results of Mori *et al.* [6], local Nusselt number depends on four parameters: b/L, δ/L, Pe and k_s/k_f. In this work, if we treat Nu_x^* as a reduced Nusselt number, then Nu_x^* depends only on two parameters besides axial distance. It should be mentioned that Δ is a lump parameter which combines the effects of Pe, k_s/k_f and b/L. In other words, the dependence of Nu_x on Pe, k_s/k_f and b/L is not individually but through a lump parameter Δ. Results for boundary condition of uniform heat flux will be presented in detail.

Figures 2 and 3 show the effect of Δ and δ/L on the interface temperature profile. From these two figures, it

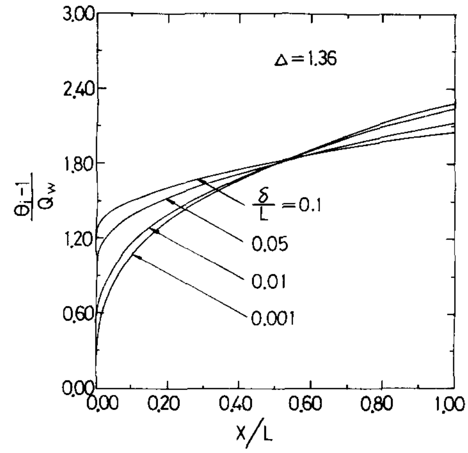

FIG. 2. Axial distribution of interfacial temperature for different values of δ/L, UHF case, $\Delta = 1.36$.

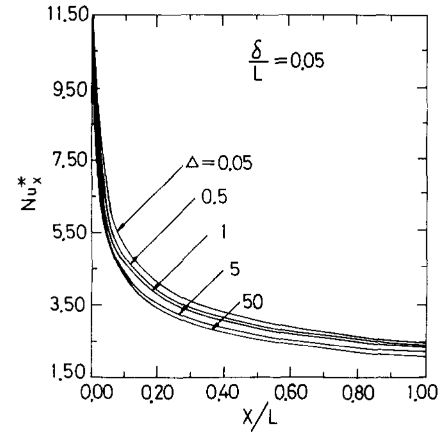

FIG. 4. Axial distribution of local Nusselt number for different values of Δ, UHF case, $\delta/L = 0.05$.

is clear that an increase in δ/L or Δ tends to make the interfacial temperature more uniform. As discussed in the previous section, since Δ is a lump parameter which combines the effects of Pe, b/L and k_s/k_f, results of this work are superior to those of previous investigators in which only the effects of individual parameter were shown.

Figures 4 and 5 show the results of reduced local Nusselt number as a function of axial distance with Δ and δ/L as parameters, respectively. Mori *et al.* [6] obtained interfacial temperature as well as local Nusselt number as function of axial distance by using the eigenfunction expansion technique. A comparison of Figs. 2 and 4 with those presented by Mori *et al.* shows close agreement except for a small discrepancy ($\simeq 5\%$) when $x^\dagger = 0.005$ which corresponds to the channel outlet in Mori *et al.*'s case. This is because at a distance far from the channel inlet, the assumption of a thin thermal boundary layer may not be valid. As long as $x^\dagger \leqslant 0.001$, the solution of this work is very accurate. It should be mentioned that for small $x^\dagger$, the eigenfunction expansion solution converges too slowly to be of practical value. In Mori *et al.*'s work, at $x^\dagger \leqslant 0.001$, up to 40 terms had to be used in the eigenfunction expansion solution. The calculation is tedious and solution may not converge. In most practical applications, except for the heat transfer involving liquid metals, the Péclet number is large enough such that $x^\dagger \leqslant 0.001$, except possibly far away from the channel inlet. The asymptotic solution presented in this work can be applied to find the local Nusselt number.

Figures 6 and 7 show the local Nusselt number as function of axial distance on log–log scale with Δ and δ/L as parameters. As expected, all results fall between the classical Leveque solutions for constant wall temperature and constant wall flux.

CONCLUDING REMARKS

The heat transfer problem of a high Prandtl number, Newtonian fluid flowing between two thick, parallel plates is solved in this work. This is a conjugate problem in which the energy equations of solid phase and liquid

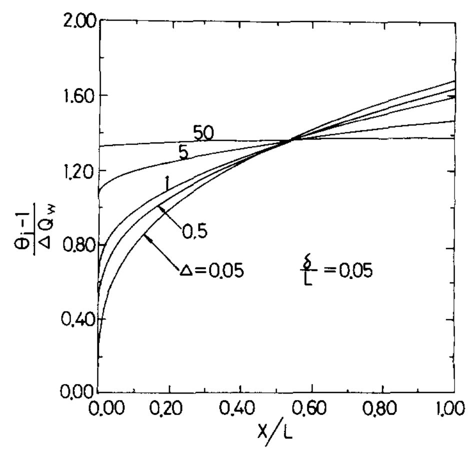

FIG. 3. Axial distribution of interfacial temperature for different values of Δ, UHF case, $\delta/L = 0.05$.

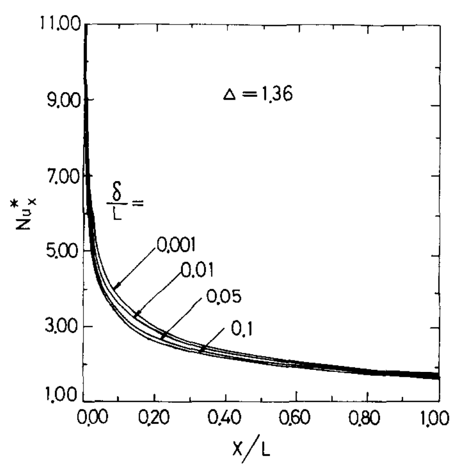

FIG. 5. Axial distribution of local Nusselt number for different values of δ/L, UHF case, $\Delta = 1.36$.

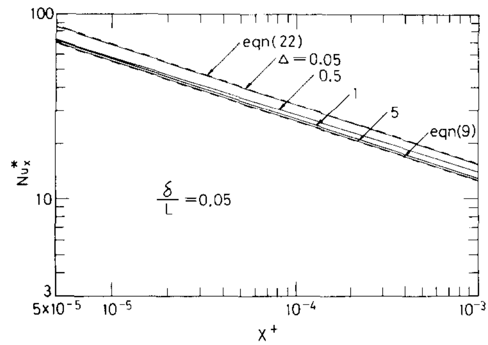

FIG. 6. Log–log plot of axial distribution of local Nusselt number for different values of Δ, UHF case, $\delta/L = 0.05$.

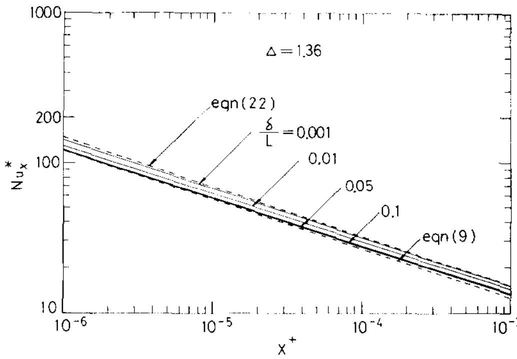

FIG. 7. Log–log plot of axial distribution of local Nusselt number for different values of δ/L, UHF case, $\Delta = 1.36$.

phase are coupled through boundary conditions at the interface. The results are characterized by two parameters: one is the ratio of the thickness of flat plate to the length of the channel and the other is a lump parameter which combines the effects of all other parameters. Although we have only treated Newtonian fluid flowing between parallel plates in this work, the same method can be applied to non-Newtonian fluid flowing in ducts of other geometries.

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SOLUTION CONJUGUEE DE LEVEQUE POUR UN FLUIDE NEWTONIEN DANS UN CANAL A PLANS PARALLELES

Résumé—On présente la solution de Lévêque pour le problème conjuguée d'un fluide newtonien à grand nombre de Prandtl qui s'écoule dans un canal à plans parallèles de longueur finie. Une procédure est proposée pour trouver la distribution approchée de la température interfaciale. On trouve que l'effet de la conduction pariétale peut être caractérisé par deux paramètres. Une solution analytique approchée du nombre de Nusselt local est obtenue. Bien que la solution présentée est supposée valable seulement pour $x^+ \leqslant 0{,}001$, les résultats ne s'écartent pas significativement de la solution connue par développement en fonctions propres, même quand $x^+ = 0{,}005$.

DIE GEKOPPELTE LEVEQUE-LÖSUNG FÜR NEWTON'SCHE FLUIDE IN EINEM RECHTECKKANAL

Zusammenfassung—Die Leveque-Lösung für ein gekoppeltes Problem bei Newton'schen Fluiden mit großer Prandtl-Zahl, die in einem Rechteckkanal von begrenzter Länge strömen, wird vorgestellt. Ein Verfahren wird vorgeschlagen, um die Grenzflächentemperatur-Verteilung näherungsweise zu ermitteln. Es ergab sich, daß der Einfluß der Wärmeleitung in der Wand mit zwei Parametern charakterisiert werden kann. Man erhält eine geschlossene Näherungslösung für die örtliche Nusselt-Zahl. Obgleich für die in dieser Arbeit vorgestellte Lösung angenommen wird, daß sie nur für $x^+ \leqslant 0{,}001$ gültig ist, zeigen die Ergebnisse, daß sich sogar für den Wert $x^+ = 0{,}005$ keine bedeutenden Abweichungen von der bekannten Lösung (Entwicklung der Eigenfunktion) ergeben.

СОПРЯЖЕННОЕ РЕШЕНИЕ ЛЕВЕКА ДЛЯ ТЕЧЕНИЯ НЬЮТОНОВСКОЙ ЖИДКОСТИ В ПАРАЛЛЕЛЬНОМ ПЛОСКОМ КАНАЛЕ

Аннотация—Получено решение Левека сопряженной задачи для течения ньютоновской жидкости с большим числом Прандтля в параллельном плоском канале конечной длины. Предложен метод нахождения приближенного распределения температуры на межфазной границе. Показано, что эффект проводимости стенки может быть охарактеризован двумя параметрами. Получено замкнутое приближенное выражение для местного числа Нуссельта. Несмотря на то, что со строгой точки зрения решение справедливо только для $x^+ \leqslant 0{,}001$, результаты показывают, что нет существенного отклонения от известного решения для разложения по собственным функциям даже при $x^+ = 0{,}005$.